Tanta University
Faculty of Engineering
Physics and Engineering Mathematics Department

Sheet –Number 1 Differential Equation

(a) Verify that y is a solution of the ODE

1. $y' + 5xy = 0, \quad y = ce^{-2.5x^2}$
2. $yy' = 4x, \quad y^2 - 4x^2 = c \ (y > 0)$
3. $y' = y - y^2, \quad y = \dfrac{1}{1 + ce^{-x}}$
4. $y' \tan x = 2y - 8, \quad y = c \sin^2 x + 4$

(b) find the solution of the following ODE

1. $y' = \sec^2 y$
2. $y' = e^{2x-1}y^2$
3. $xy' = y + 2x^3 \sin^2 \dfrac{y}{x} \quad (\text{Set } y/x = u)$
4. $y' = (y + 4x)^2 \quad (\text{Set } y + 4x = v)$
5. $y' = 1 + 4y^2, \quad y(1) = 0$
6. $y' \cosh^2 x = \sin^2 y, \quad y(0) = \frac{1}{2}\pi$
7. $(\text{Set } v = x + y - 2)\ y' = (x + y - 2)^2, \quad y(0) = 2$
8. $(\text{Set } y/x = u)\ \ xy' = y + 3x^4 \cos^2 (y/x), \quad y(1) = 0$
9. $\sin x \cos y \, dx + \cos x \sin y \, dy = 0$
10. $e^{3\theta}(dr + 3r \, d\theta) = 0$
11. $(x^2 + y^2) \, dx - 2xy \, dy = 0$
12. $2x \tan y \, dx + \sec^2 y \, dy = 0$
13. $e^{2x}(2 \cos y \, dx - \sin y \, dy) = 0, \quad y(0) = 0$
14. $(2xy \, dx + dy)e^{x^2} = 0, \quad y(0) = 2$
15. $y' = 2y - 4x$
16. $y' + 2y = 4 \cos 2x, \quad y(\frac{1}{4}\pi) = 3$
17. $xy' = 2y + x^3 e^x$
18. $y' + y \tan x = e^{-0.01x} \cos x, \quad y(0) = 0$
19. $y' + y \sin x = e^{\cos x}, \quad y(0) = -2.5$
20. $y' \cos x + (3y - 1) \sec x = 0, \quad y(\frac{1}{4}\pi) = 4/3$
21. $xy' + 4y = 8x^4, \quad y(1) = 2$
22. $y' + y = y^2, \quad y(0) = -\frac{1}{3}$
23. $y' + xy = xy^{-1}, \quad y(0) = 3$
24. $y' = (\tan y)/(x - 1), \quad y(0) = \frac{1}{2}\pi$
25. $y' = 1/(6e^y - 2x)$
26. $2xyy' + (x - 1)y^2 = x^2 e^x \quad (\text{Set } y^2 = z)$
27. $(3xe^y + 2y) \, dx + (x^2 e^y + x) \, dy = 0$
28. $y' = \sqrt{1 - y^2}, \quad y(0) = 1/\sqrt{2}$
29. $y' + 4xy = e^{-2x^2}, \quad y(0) = -4.3$
30. $y' = ay + by^2 \quad (a \neq 0)$

This sheet is extracted from Advanced Engineering Mathematics Text Book

Differentiation

$(cu)' = cu'$ (c constant)

$(u + v)' = u' + v'$

$(uv)' = u'v + uv'$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx} \qquad \text{(Chain rule)}$$

$(x^n)' = nx^{n-1}$

$(e^x)' = e^x$

$(e^{ax})' = ae^{ax}$

$(a^x)' = a^x \ln a$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\tan x)' = \sec^2 x$

$(\cot x)' = -\csc^2 x$

$(\sinh x)' = \cosh x$

$(\cosh x)' = \sinh x$

$$(\ln x)' = \frac{1}{x}$$

$$(\log_a x)' = \frac{\log_a e}{x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1 - x^2}}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1 - x^2}}$$

$$(\arctan x)' = \frac{1}{1 + x^2}$$

$$(\operatorname{arccot} x)' = -\frac{1}{1 + x^2}$$

Integration

$$\int uv'\,dx = uv - \int u'v\,dx \text{ (by parts)}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \qquad (n \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + c$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$

$$\int \sin x\,dx = -\cos x + c$$

$$\int \cos x\,dx = \sin x + c$$

$$\int \tan x\,dx = -\ln|\cos x| + c$$

$$\int \cot x\,dx = \ln|\sin x| + c$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + c$$

$$\int \csc x\,dx = \ln|\csc x - \cot x| + c$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \operatorname{arcsinh}\frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \operatorname{arccosh}\frac{x}{a} + c$$

$$\int \sin^2 x\,dx = \tfrac{1}{2}x - \tfrac{1}{4}\sin 2x + c$$

$$\int \cos^2 x\,dx = \tfrac{1}{2}x + \tfrac{1}{4}\sin 2x + c$$

$$\int \tan^2 x\,dx = \tan x - x + c$$

$$\int \cot^2 x\,dx = -\cot x - x + c$$

$$\int \ln x\,dx = x\ln x - x + c$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx) + c$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a\cos bx + b\sin bx) + c$$